Published by Institute of Physics Publishing for SISSA/ISAS

Received: *September 7, 2004*
Revised: *October 5, 2004*
Accepted: *October 5, 2004*

Superfield description of 5D supergravity on general warped geometry

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Abstract: We provide a systematic and practical method of deriving 5D supergravity action described by 4D superfields on a *general warped geometry*, including a non-BPS background. Our method is based on the superconformal formulation of 5D supergravity, but is easy to handle thanks to the superfield formalism. We identify the radion superfield in the language of 5D superconformal gravity, and clarify its appearance in the action. We also discuss SUSY breaking effects induced by a deformed geometry due to the backreaction of the radius stabilizer.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Models.

Contents

1. Introduction

In recent years, the brane-world scenario [1, 2] has attracted much attention as a candidate for the new physics beyond the standard model and has been investigated in various frameworks. The idea that our four-dimensional (4D) world is embedded into a higher dimensional spacetime was introduced[1] by Hořava and Witten in their investigation of the strongly-coupled heterotic string theory [4]. Five-dimensional supergravity (5D SUGRA) on an orbifold S^1/Z_2 appears as a low-energy effective theory after the reduction of their theory to five dimensions by compactifying on a Calabi-Yau 3-fold [5]. In addition, 5D SUGRA has been investigated actively as a simplest stage of the supersymmetric (SUSY) brane-world scenario.

[1] In the context of field theory, such an idea was originally studied in ref. [3].

When we construct brane-world models, a size of the extra dimensions has to be stabilized. One of the main stabilization mechanisms is proposed in ref. [6], and similar mechanisms have also been studied [7]–[9]. These mechanisms involve a bulk scalar field that has a nontrivial field configuration. Generically, such a scalar configuration does not saturate Bogomol'nyi-Prasad-Sommerfield (BPS) bound [10], and breaks SUSY [8]. At the same time, the background geometry receives the backreaction of the scalar configuration, which has not been taken into account in most works. Thus, SUSY breaking effects can be mediated to our visible sector through the deformation of the spacetime geometry even if the visible sector is decoupled from the stabilization sector where SUSY breaking occurs. One of the authors discussed such SUSY breaking effects in a toy model where the bulk spacetime is four dimensions and the effective theory is three-dimensional [11]. In that work, a systematic method is proposed to describe a 4D SUGRA action in terms of 3D superfields on a *general warped geometry*, including a non-BPS background. In this paper, we will apply this method to 5D SUGRA and derive 5D SUGRA action described by 4D superfields on the general warped geometry.

For concreteness of the discussion, we will assume that the radius of the extra dimension is stabilized by a scalar configuration of a hypermultiplet, which is decoupled from the visible sector. In this case, the spacetime geometry is determined by the nontrivial vacuum configuration of the radius stabilizer in the hidden sector besides the cosmological constant through the Einstein equation. We will also assume that the extra dimension is compactified on an orbifold S^1/Z_2. In this paper, a characteristic scale of the hidden sector $\Lambda_{\rm hid}$ is supposed to be much smaller than the 5D Planck mass M_5. For example, $\Lambda_{\rm hid}$ is supposed to be an intermediate scale. Thus, interactions with the gravitational fields can be neglected. Especially, in the limit of $\Lambda_{\rm hid}/M_5 \to 0$, the deformation of the geometry vanishes and the spacetime becomes flat or a slice of anti-de Sitter (AdS) space.

The superconformal formulation of 5D SUGRA is a systematic and elegant way of describing the bulk-boundary system [12]–[15], but it involves very complicated and tedious calculations to derive the action. Our method is based on the superconformal formulation, but is much easier to handle thanks to its 4D superfield description. Thus, our method is useful as a practical method of deriving the 4D effective action expressed by superfields.

The radion superfield plays an important role in transmission of SUSY breaking effects to our visible sector [16, 17]. Although there are some papers identifying the radion chiral multiplet with fields in the off-shell 5D SUGRA [18], no one has constructed the radion *superfield* from the off-shell SUGRA directly. Using our method, we can identify the radion superfield and see its appearance in the action directly from the off-shell SUGRA.

The paper is organized as follows. In the next section, we will provide an invariant action expressed by 4D superfields. In section 3, we will explain the gauge fixing of the extraneous superconformal symmetry, and specify the physical superfields. In section 4, we will derive the action after the gauge fixing, and identify the radion superfield. SUSY breaking induced by the geometry is also discussed. Section 5 is devoted to the summary. The notation we use in this paper is listed in appendix A, and brief comments on the invariant action is provided in appendix B.

2. Off-shell formulation

In this section, we will briefly explain the field content in the superconformal formulation of 5D SUGRA. Then, we will construct 4D superfields[2] and express an invariant action in terms of them. Basically, we will follow the notation of ref. [14]. Throughout this paper, we will use $\mu, \nu, \cdots = 0,1,2,3,4$ for the 5D world vector indices, and $m, n, \cdots = 0,1,2,3$ for the 4D indices. The coordinate of an extra dimension is denoted as $y \equiv x^4$. The corresponding local Lorentz indices are denoted by underbarred indices.

2.1 Field contents

The 5D Weyl multiplet consists of the following fields.

$$e_\mu{}^{\underline{\nu}}\,, \qquad \psi^i_\mu\,, \qquad V^{ij}_\mu \qquad b_\mu\,, \qquad v_{\underline{\mu}\underline{\nu}}\,, \qquad \chi^i\,, \qquad D\,, \tag{2.1}$$

where $i, j = 1, 2$ are indices of $\mathrm{SU}(2)_{\boldsymbol{U}}$, which is related to $\mathrm{SU}(2)_R$ after the superconformal gauge fixing. These are the fünfbein, the gravitino, the gauge bosons for $\mathrm{SU}(2)_{\boldsymbol{U}}$ and the dilatation, a real antisymmetric tensor, an $\mathrm{SU}(2)_{\boldsymbol{U}}$ Majorana spinor, and a real scalar, respectively. Among these, only $e_\mu{}^{\underline{\nu}}$ and ψ^i_μ are dynamical. b_μ is eliminated by the gauge fixing condition, and the remaining ones are auxiliary fields. Especially, χ^i and D appear in the action in the form of Lagrange multipliers [13].

In this paper, we will introduce vector multiplets $\mathcal{V}^I$ $(I = 1, 2, \ldots, n_{\mathrm{V}})$ and hypermultiplets $\mathcal{H}^{\hat{\alpha}}$ $(\hat{\alpha} = 1, 2, \ldots, n_{\mathrm{H}})$ as matter multiplets. In addition to these, there is a vector multiplet $\mathcal{V}^0$ that includes the graviphoton,[3] and a compensator hypermultiplet $\mathcal{H}^0$.[4] Thus, there are $n_{\mathrm{V}} + 1$ vector multiplets $\mathcal{V}^I$ $(I = 0, 1, 2, \ldots, n_{\mathrm{V}})$, and $n_{\mathrm{H}} + 1$ hypermultiplets $\mathcal{H}^{\hat{\alpha}}$ $(\hat{\alpha} = 0, 1, 2, \ldots, n_{\mathrm{H}})$.

A vector multiplet $\mathcal{V}^I$ consists of

$$M^I\,, \qquad W^I_\mu\,, \qquad \Omega^{Ii}\,, \qquad Y^{Iij}\,, \tag{2.2}$$

which are a gauge scalar, a gauge field, a gaugino and an auxiliary field, respectively.

The hypermultiplets consist of complex scalars $\mathcal{A}^\alpha_i$, spinors ζ^α and auxiliary fields $\mathcal{F}^\alpha_i$. They carry a $Usp(2, 2n_{\mathrm{H}})$ index α $(\alpha = 1, 2, \ldots, 2n_{\mathrm{H}} + 2)$ on which the gauge group G can act. These are split into $n_{\mathrm{H}} + 1$ hypermultiplets as

$$\mathcal{H}^{\hat{\alpha}} = \left(\mathcal{A}^{2\hat{\alpha}+1}_i, \mathcal{A}^{2\hat{\alpha}+2}_i, \zeta^{2\hat{\alpha}+1}, \zeta^{2\hat{\alpha}+2}, \mathcal{F}^{2\hat{\alpha}+1}_i, \mathcal{F}^{2\hat{\alpha}+2}_i\right). \tag{2.3}$$

As mentioned above, $\mathcal{H}^{\hat{\alpha}=0}$ is used as a compensator multiplet. $\mathcal{H}^{\hat{\alpha}=1}$ is supposed to be a stabilizer multiplet whose scalar components have a nontrivial vacuum configuration that is relevant to the radius stabilization. The remaining hypermultiplets are matter fields in the visible sector.

[2]In this paper, we will use the word "4D superfield" for an $\mathcal{N} = 1$ superfield that is allowed to have y-dependence.

[3]In ref. [14], this is called the central charge vector multiplet.

[4]We will assume that there is only one compensator hypermultiplet. An extension to the multi-compensator case is straightforward.

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Weyl multiplet	
$\Pi=+1$	$e_m{}^{\underline{n}}, e_y{}^4, \psi^1_{m\mathrm{R}}, \psi^2_{y\mathrm{R}}, b_m, V_m^{(3)}, V_y^{(1,2)}, v_{4\underline{m}}, \chi^1_{\mathrm{R}}, D$
$\Pi=-1$	$e_m{}^4, e_y{}^{\underline{n}}, \psi^2_{m\mathrm{R}}, \psi^1_{y\mathrm{R}}, b_y, V_y^{(3)}, V_m^{(1,2)}, v_{\underline{mn}}, \chi^2_{\mathrm{R}}$
Vector multiplet	
Π_I	$W^I_m, \quad \Omega^{I1}_{\mathrm{R}}, \quad Y^{I(3)}$
$-\Pi_I$	$M^I, \quad W^I_y, \quad \Omega^{I2}_{\mathrm{R}}, \quad Y^{I(1,2)}$
Hypermultiplet	
$\Pi=+1$	$\mathcal{A}^{2\hat{\alpha}+2}_{i=2}, \quad \mathcal{F}^{2\hat{\alpha}+2}_{i=1}, \quad \zeta^{2\hat{\alpha}+2}_{\mathrm{R}}$
$\Pi=-1$	$\mathcal{A}^{2\hat{\alpha}+1}_{i=2}, \quad \mathcal{F}^{2\hat{\alpha}+1}_{i=1}, \quad \zeta^{2\hat{\alpha}+1}_{\mathrm{R}}$

Table 1: Orbifold parity eigenvalues.

The hyperscalars $\mathcal{A}^\alpha_i$ satisfy the hermiticity condition

$$\mathcal{A}^i_\alpha \equiv \epsilon^{ij}\mathcal{A}^\beta_j \rho_{\beta\alpha} = -(\mathcal{A}^\alpha_i)^*\,, \tag{2.4}$$

where $\rho_{\alpha\beta}$ is an antisymmetric tensor (see appendix A). Thus, we can choose $\mathcal{A}^\alpha_{i=2}$ ($\alpha = 1,2,\ldots,2n_{\mathrm{H}}+2$) as independent fields. The auxiliary fields $\mathcal{F}^\alpha_i$ also satisfy the same type of condition.

The gravitino ψ^i_μ and the gauginos Ω^{Ii} are SU$(2)_{\boldsymbol{U}}$ Majorana spinors, and the hyperinos ζ^α are $Usp(2,2n_{\mathrm{H}})$ Majorana spinors. Due to the corresponding Majorana conditions, we can choose only the right-handed components of these spinors as independent fields. Thus, we will use a 2-component spinor notation defined by eq. (A.7) in the following sections. The relation to the 4-component spinor notation is referred in appendix A.

V^{ij}_μ and Y^{Iij} are triplets of SU$(2)_{\boldsymbol{U}}$. In the following, we will use the 'isovector' notation for these triplets, i.e.,

$$V^{ik}_\mu \epsilon_{kj} \equiv i\sum_{r=1}^{3} V^{(r)}_\mu (\sigma_r)^i{}_j\,, \qquad Y^{Iik}\epsilon_{kj} \equiv i\sum_{r=1}^{3} Y^{I(r)}(\sigma_r)^i{}_j\,, \tag{2.5}$$

where ϵ_{ij} is an antisymmetric tensor ($\epsilon_{12}=1$), and σ_r ($r=1,2,3$) are the Pauli matrices.

The orbifold parity of each field is listed in table 1. Here, we will assume that all the physical vector multiplets have massless 4D vector fields in the Kaluza-Klein decomposition, while the graviphoton has an odd parity and no massless mode. Namely, $\Pi_{I=0}=-1$ and $\Pi_{I\neq 0}=+1$. For the hypermultiplets, we can always redefine the fields by using a degree of freedom for $Usp(2,2n_{\mathrm{H}})$ so that their parity eigenvalues are those listed in the table.

2.2 Superfields and invariant action

We will take the warped metric ansatz for the background geometry,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{2\sigma(y)}\eta_{\underline{mn}}dx^m dx^n - \langle e_y{}^4\rangle^2 dy^2\,, \tag{2.6}$$

where $\langle e_y{}^4\rangle$ is a vacuum expectation value (VEV) of $e_y{}^4$.

Since we are not interested in the gravitational interactions, which are suppressed by the large Planck mass M_5, we will freeze the gravitational multiplet to its background value.[5]

$$\begin{aligned}
\langle e_{\underline{m}}{}^{\underline{n}} \rangle &= e^{\sigma} \delta_{\underline{m}}{}^{\underline{n}} \,, \qquad \langle e_y{}^4 \rangle = \text{constant} \,, \\
\langle \psi_\mu^i \rangle &= 0 \,, \\
\langle V_m^{(r)} \rangle &= 0, \qquad (r = 1,2,3) \\
\langle v_{\underline{\mu\nu}} \rangle &= 0 \,,
\end{aligned} \tag{2.7}$$

where $\langle \cdots \rangle$ denotes a VEV of the argument. Note that ψ_μ^i, $V_m^{(r)}$ and $v_{\underline{\mu\nu}}$ cannot have non-zero VEVs because of the unbroken 4D Poincaré invariance. On the other hand, the extra-components of the $\mathrm{SU}(2)_{\boldsymbol{U}}$ gauge fields $V_y^{(r)}$ can have non-zero VEVs. To simplify the discussion, we will ignore the dependence on $\langle V_y^{(3)} \rangle$. We will comment on this point in section 4.4. In the following, we will also drop the dependences on b_μ, χ^i and D because b_μ will be set to zero by the gauge fixing condition and χ^i and D only play Lagrange multipliers as mentioned in the previous subsection. We can always rescale the coordinate y so that $\langle e_y{}^4 \rangle = 1$, but here we will leave $\langle e_y{}^4 \rangle$ as an arbitrary constant in order to make clear the correspondence to the 4D superconformal multiplets in ref. [14].

In the case of $\langle V_y^{(r)} \rangle = 0$, we can construct 4D superfields from 5D vector and hyper multiplets, and express an invariant action on the above gravitational background in terms of them by using a method proposed in ref. [11]. By introducing the spurion superfield[6]

$$V_T = \langle e_y{}^4 \rangle + i\theta^2 e^{\sigma} \langle V_y^{(1)} + iV_y^{(2)} \rangle - i\bar{\theta}^2 e^{\sigma} \langle V_y^{(1)} - iV_y^{(2)} \rangle \,, \tag{2.8}$$

and modifying some auxiliary components of the above constructed superfields, we can incorporate $\langle V_y^{(1)} \rangle$ and $\langle V_y^{(2)} \rangle$ into the invariant action. Each superfield is defined as follows.

From the vector multiplets, we will define the following 4D vector and chiral superfields.

$$\begin{aligned}
V^I &\equiv \theta \sigma^{\underline{m}} \bar{\theta} W_m^I + i\theta^2 \bar{\theta} \bar{\lambda}^I - i\bar{\theta}^2 \theta \lambda^I + \frac{1}{2} \theta^2 \bar{\theta}^2 D^I \,, \\
\Phi_S^I &\equiv \varphi_S^I - \theta \chi_S^I - \theta^2 \mathcal{F}_S^I \,,
\end{aligned} \tag{2.9}$$

where[7]

$$\begin{aligned}
\lambda^I &\equiv 2e^{\frac{3}{2}\sigma} \Omega_{\mathrm{R}}^{I1} \,, \\
D^I &\equiv -e^{2\sigma} \left\{ \langle e_y{}^4 \rangle^{-1} \partial_y M^I - 2Y^{I(3)} + \langle e_y{}^4 \rangle^{-1} \dot{\sigma} M^I \right\} \,, \\
\varphi_S^I &\equiv \frac{1}{2} \left(W_y^I + i \langle e_y{}^4 \rangle M^I \right) \,, \\
\chi_S^I &\equiv -2e^{\frac{\sigma}{2}} \langle e_y{}^4 \rangle \Omega_{\mathrm{R}}^{I2} \,, \\
\mathcal{F}_S^I &\equiv e^{\sigma} \left\{ \langle V_y^{(1)} + iV_y^{(2)} \rangle M^I - i \langle e_y{}^4 \rangle \left(Y^{I(1)} + iY^{I(2)} \right) \right\} \,.
\end{aligned} \tag{2.10}$$

[5]The replacement of $v_{\underline{\mu\nu}}$ with its background value causes short of some terms in the invariant action, but this is not a problem for our purpose. (See appendix B.)

[6]This superfield corresponds to the 4D general multiplet $\boldsymbol{W}_y$ in ref. [14].

[7]The minus signs of the coefficients of θ and θ^2 in the chiral superfields are necessary for matching to the notation of refs. [13, 14, 15].

The dot denotes derivative in terms of y. Here, we have chosen the Wess-Zumino gauge. For simplicity, we will consider only abelian gauge groups in this paper. Under the gauge transformation, the above superfields transform as

$$\begin{aligned} V^I &\to V^I + \Lambda^I + \bar{\Lambda}^I , \\ \Phi_S^I &\to \Phi_S^I + i\partial_y \Lambda^I , \end{aligned} \tag{2.11}$$

where Λ^I are arbitrary chiral superfields. Thus, the gauge invariant quantities are

$$\begin{aligned} V_S^I &\equiv \frac{\left(-\partial_y V^I - i\Phi_S^I + i\bar{\Phi}_S^I\right)}{V_T} , \\ \mathcal{W}_\alpha^I &\equiv -\frac{1}{4}\bar{D}^2 D_\alpha V^I . \end{aligned} \tag{2.12}$$

Here, V_T in the definition of V_S^I is not necessary for the invariance under the transformation (2.11), but is necessary for covariantization of y-derivative in terms of $\mathrm{SU}(2)_{\boldsymbol{U}}$.

From the hypermultiplets, we will define the following chiral superfields.

$$\Phi^\alpha \equiv \varphi^\alpha - \theta\chi^\alpha - \theta^2 \mathcal{F}^\alpha , \tag{2.13}$$

where

$$\begin{aligned} \varphi^\alpha &\equiv \mathcal{A}_2^\alpha , \\ \chi^\alpha &\equiv -2ie^{\frac{\sigma}{2}}\zeta_{\mathrm{R}}^\alpha , \\ \mathcal{F}^\alpha &\equiv e^\sigma \langle e_y{}^4\rangle^{-1}\Bigg\{\partial_y \mathcal{A}_1^\alpha + i\langle V_y^{(1)} + iV_y^{(2)}\rangle \mathcal{A}_2^\alpha + i\left(\langle e_y{}^4\rangle + \frac{iW_y^0}{M^0}\right)\tilde{\mathcal{F}}_1^\alpha - \\ &\qquad - \left(W_y^I - i\langle e_y{}^4\rangle M^I\right)(gt_I)^\alpha{}_\beta \mathcal{A}_1^\beta + \frac{3}{2}\dot{\sigma}\mathcal{A}_1^\alpha\Bigg\} . \end{aligned} \tag{2.14}$$

In the last expression, $\tilde{\mathcal{F}}_1^\alpha$ is defined as

$$\tilde{\mathcal{F}}_1^\alpha \equiv \mathcal{F}_1^\alpha - M^0(gt_0)^\alpha{}_\beta \mathcal{A}_1^\beta . \tag{2.15}$$

The generators of the gauge group t_I $(I = 0, 1, \ldots, n_{\mathrm{V}})$ are defined as anti-hermitian.

Using these superfields, the 5D invariant action can be written as follows.

$$\begin{aligned} S &= \int d^5x\ \mathcal{L}_{\mathrm{vector}} + \mathcal{L}_{\mathrm{hyper}} , \\ \mathcal{L}_{\mathrm{vector}} &= \left[\int d^2\theta\ \frac{3C_{IJK}}{2}\left\{i\Phi_S^I \mathcal{W}^J \mathcal{W}^K + \frac{1}{12}\bar{D}^2\left(V^I D^\alpha \partial_y V^J - D^\alpha V^I \partial_y V^J\right)\mathcal{W}_\alpha^K\right\} + \mathrm{h.c.}\right] - \\ &\quad - e^{2\sigma}\int d^4\theta\ V_T C_{IJK} V_S^I V_S^J V_S^K , \\ \mathcal{L}_{\mathrm{hyper}} &= -2e^{2\sigma}\int d^4\theta\ V_T d_\alpha{}^\beta \bar{\Phi}^\beta \left(e^{-2igV^I t_I}\right)^\alpha{}_\gamma \Phi^\gamma - \\ &\quad - e^{3\sigma}\left[\int d^2\theta\ \Phi^\alpha d_\alpha{}^\beta \rho_{\beta\gamma}\left(\partial_y - 2g\Phi_S^I t_I\right)^\gamma{}_\delta \Phi^\delta + \mathrm{h.c.}\right] , \end{aligned} \tag{2.16}$$

where C_{IJK} is a real constant tensor which is completely symmetric for the indices, and $d_\alpha{}^\beta$ is a metric of the hyperscalar space and can be brought into the standard form [28]

$$d_\alpha{}^\beta = \begin{pmatrix} \mathbf{1}_2 & \\ & -\mathbf{1}_{2n_{\rm H}} \end{pmatrix} . \tag{2.17}$$

Eq. (2.16) is very similar to the action in ref. [19], except for the appearance of the spurion superfield V_T and the warp factor.[8] However, note that our action is SUGRA action *before the superconformal gauge fixing*, which is explained in the next section. The above action certainly reproduces the component action in refs. [13, 15].[9] (See appendix B.) The first line of $\mathcal{L}_{\rm vector}$ corresponds to the gauge kinetic terms and the supersymmetric Chern-Simons term. In fact, variation of these terms under the gauge transformation (2.11) does not vanish, but becomes total derivative. Thus, these terms cannot be expressed by only the gauge-invariant quantities V_S^I and $\mathcal{W}^I$.

In addition to the above action, we can also introduce the brane actions localized at the fixed boundaries of the orbifold. We will discuss them in Sect. 4.1.

3. Gauge fixing and physical superfields

3.1 Gauge fixing conditions

In order to obtain the Poincaré supergravity, we have to fix the extraneous superconformal symmetries, i.e., the dilatation $\boldsymbol{D}$, $\mathrm{SU}(2)_{\boldsymbol{U}}$, the conformal supersymmetry $\boldsymbol{S}$ and the special conformal transformation $\boldsymbol{K}$. The gauge fixing condition for each symmetry is as follows [14, 15].

The $\boldsymbol{D}$-gauge is fixed by

$$\mathcal{N} \equiv C_{IJK} M^I M^J M^K = M_5^3 \,, \tag{3.1}$$

$$\mathcal{A}_i^\alpha d_\alpha{}^\beta \mathcal{A}_\beta^i = 2\left\{ -\sum_{a=1}^{2} |\mathcal{A}_2^a|^2 + \sum_{\underline{\alpha}=3}^{2n_{\rm H}+2} \left|\mathcal{A}_2^{\underline{\alpha}}\right|^2 \right\} = -2M_5^3 \,, \tag{3.2}$$

where $\mathcal{N}$ is called a norm function, and $\mathrm{SU}(2)_{\boldsymbol{U}}$ is fixed by the condition

$$\mathcal{A}_i^a \propto \delta_i^a \,, \qquad (a = 1, 2) \,. \tag{3.3}$$

The $\boldsymbol{S}$-gauge is fixed by

$$\mathcal{N}_I \Omega^{Ii} = 0 \,, \tag{3.4}$$

$$\mathcal{A}_i^\alpha d_\alpha{}^\beta \zeta_\beta = 0 \,, \tag{3.5}$$

where $\mathcal{N}_I \equiv \partial\mathcal{N}/\partial M^I$.

[8] The V_T-dependence in eq. (2.16) is also taken into account in ref. [20].

[9] An invariant action constructed in ref. [13] coincides with that of ref. [15] on the gravitational background (2.7), although $\boldsymbol{S}$-gauge is already fixed from the beginning in ref. [13].

The $\boldsymbol{K}$-gauge fixing condition is

$$b_\mu = 0\,, \tag{3.6}$$

which is already taken into account in our case.

Here, note that the compensator scalars $\mathcal{A}_i^a$ $(a = 1, 2)$ are charged under both $\mathrm{SU}(2)_{\boldsymbol{U}}$ and an SU(2) subgroup of $Usp(2, 2n_\mathrm{H})$ that rotates only the compensator part, which refer to as $\mathrm{SU}(2)_C$. Thus, the $\mathrm{SU}(2)_{\boldsymbol{U}}$ gauge fixing condition (3.3) breaks $\mathrm{SU}(2)_{\boldsymbol{U}} \times \mathrm{SU}(2)_C$ up to an SU(2) diagonal subgroup. This is the symmetry known as $\mathrm{SU}(2)_R$ in the Poincaré supergravity.

In the original superconformal formulation, the conditions (3.1) and (3.2), or (3.4) and (3.5) are not independent because the equations of motion for χ^i and D in the Weyl multiplet lead to the constraints

$$\begin{aligned} \mathcal{A}_i^\alpha d_\alpha{}^\beta \mathcal{A}_\beta^i + 2\mathcal{N} &= 0\,, & (3.7)\\ \mathcal{A}_i^\alpha d_\alpha{}^\beta \zeta_\beta + \mathcal{N}_I \Omega_i^I &= 0\,. & (3.8) \end{aligned}$$

In our case, however, we must impose these constraints by hand since we have dropped the dependences of χ^i and D.

In the vector sector, we will consider the maximally symmetric case in the following. Namely, the norm function is

$$\mathcal{N} = (M^{I=0})^3 - \frac{1}{2} M^{I=0} \sum_{J=1}^{n_\mathrm{V}} (M^J)^2\,. \tag{3.9}$$

In this case, the gauge scalars M^I which satisfy the constraint (3.1) are parametrized by the physical scalar fields ϕ^x $(x = 1, \ldots, n_\mathrm{V})$ as follows.

$$\begin{aligned} M^{I=0} &= M_5 \left\{\cosh\hat\phi^1 \cosh\hat\phi^2 \cdots \cosh\hat\phi^{n_\mathrm{V}}\right\}^{2/3} = M_5 + \frac{1}{3M_5^2}\sum_{x=1}^{n_\mathrm{V}} (\phi^x)^2 + \mathcal{O}(M_5^{-5})\,,\\ M^{I=1} &= \frac{\sqrt{2} M_5 \sinh\hat\phi^1}{\left\{\cosh\hat\phi^1 \cosh\hat\phi^2 \cdots \cosh\hat\phi^{n_\mathrm{V}}\right\}^{1/3}} = \frac{\sqrt{2}}{M_5^{1/2}}\phi^1 + \mathcal{O}(M_5^{-7/2})\,,\\ M^{I=2} &= \frac{\sqrt{2} M_5 \cosh\hat\phi^1 \sinh\hat\phi^2}{\left\{\cosh\hat\phi^1 \cosh\hat\phi^2 \cdots \cosh\hat\phi^{n_\mathrm{V}}\right\}^{1/3}} = \frac{\sqrt{2}}{M_5^{1/2}}\phi^2 + \mathcal{O}(M_5^{-7/2})\,,\\ &\vdots\\ M^{I=n_\mathrm{V}} &= \frac{\sqrt{2} M_5 \cosh\hat\phi^1 \cosh\hat\phi^2 \cdots \cosh\hat\phi^{n_\mathrm{V}-1} \sinh\hat\phi^{n_\mathrm{V}}}{\left\{\cosh\hat\phi^1 \cosh\hat\phi^2 \cdots \cosh\hat\phi^{n_\mathrm{V}}\right\}^{1/3}} = \frac{\sqrt{2}}{M_5^{1/2}}\phi^{n_\mathrm{V}} + \mathcal{O}(M_5^{-7/2})\,, \quad (3.10) \end{aligned}$$

where $\hat\phi^x \equiv \phi^x / M_5^{3/2}$.

In the language of 4D superfields in the previous section, the gauge fixing conditions (3.2)–(3.5) can be written as follows.

$$\begin{aligned}
\varphi^{\alpha=1} &= 0\,, \\
\varphi^{\alpha=2} &= \left(M_5^3 + \sum_{\underline{\alpha}=3}^{2n_{\rm H}+2} |\varphi^{\underline{\alpha}}|\right)^{1/2} = M_5^{3/2} + \frac{1}{2M_5^{3/2}}\sum_{\underline{\alpha}=3}^{2n_{\rm H}+2}|\varphi^{\underline{\alpha}}|^2 + \mathcal{O}(M_5^{-9/2})\,, \\
\chi^{\alpha=1} &= -\frac{1}{M_5^{3/2}}\sum_{\underline{\beta},\underline{\gamma}=3}^{2n_{\rm H}+2}\rho_{\underline{\beta}\underline{\gamma}}\varphi^{\underline{\beta}}\chi^{\underline{\gamma}} + \mathcal{O}(M_5^{-9/2})\,, \\
\chi^{\alpha=2} &= \frac{1}{M_5^{3/2}}\sum_{\underline{\beta}=3}^{2n_{\rm H}+2}\bar{\varphi}^{\underline{\beta}}\chi^{\underline{\beta}} + \mathcal{O}(M_5^{-9/2})\,, \\
\lambda^{I=0} &= \frac{\sqrt{2}}{3M_5^{3/2}}\sum_{x=1}^{n_{\rm V}}\phi^x\lambda^{I=x} + \mathcal{O}(M_5^{-9/2})\,, \\
\chi_S^{I=0} &= \frac{\sqrt{2}}{3M_5^{3/2}}\sum_{x=1}^{n_{\rm V}}\phi^x\chi_S^{I=x} + \mathcal{O}(M_5^{-9/2})\,. \qquad (3.11)
\end{aligned}$$

3.2 Physical gauge superfields

Due to the gauge fixing conditions (3.1) and (3.4), the physical gauge scalars and gauginos in 5D vector multiplets have only $n_{\rm V}$ components, respectively. Thus, the $n_{\rm V}+1$ vector multiplets can be expressed by $n_{\rm V}$ physical vector and chiral superfields $\tilde{V}^x$ and $\tilde{\Phi}_S^x$ ($x = 1,\ldots,n_{\rm V}$) besides the graviphoton B_μ after the gauge fixing. A manifold $\mathcal{M}$ of the physical gauge scalars ϕ^x is the very special manifold. Various geometrical quantities of $\mathcal{M}$ are as follows [21].

$$\begin{aligned}
h^I(\phi) &\equiv -\sqrt{\frac{2}{3}}M^I(\phi)\,, \qquad h_I(\phi) \equiv -\frac{1}{\sqrt{6}}\mathcal{N}_I\,, \qquad h_x^I(\phi) \equiv \frac{\partial M^I}{\partial\phi^x}\,, \\
a_{IJ} &\equiv -\frac{1}{2}\frac{\partial^2}{\partial M^I\partial M^J}\ln\mathcal{N}\,, \qquad a^{IJ} \equiv (a^{-1})^{IJ}\,, \\
g_{xy} &\equiv a_{IJ}h_x^Ih_y^J\,, \qquad g^{xy} \equiv (g^{-1})^{xy}\,, \qquad (3.12)
\end{aligned}$$

where a_{IJ} is a metric of the ambient $n_{\rm V}+1$ dimensional space, and g_{xy} is an induced metric on $\mathcal{M}$.

From eq. (3.10), the physical gauge scalars can be expressed as

$$\phi^x = \frac{M_5^{1/2}}{\sqrt{2}}M^{I=x} + \mathcal{O}\left(M_5^{-5/2}\right). \qquad (3.13)$$

Due to the gauge fixing condition (3.4), $h_I\lambda^I = h_I\chi_S^I = 0$. Thus, these combinations should be interpreted as the fermionic components of the graviphoton multiplet whose only nonvanishing component is the graviphoton B_μ. Note that $h_I^x \equiv g^{xy}a_{IJ}h_y^J$ are orthogonal to h_I. Thus, the physical fermions are specified as

$$\begin{aligned}
\tilde{\lambda}^x &\equiv h_I^x \lambda^I = \frac{M_5^{1/2}}{\sqrt{2}} \lambda^{I=x} + \mathcal{O}\left(M_5^{-5/2}\right), \\
\tilde{\chi}_S^x &\equiv h_I^x \chi_S^I = \frac{M_5^{1/2}}{\sqrt{2}} \chi_S^{I=x} + \mathcal{O}\left(M_5^{-5/2}\right).
\end{aligned} \tag{3.14}$$

Therefore, we can express the physical superfields in terms of the original gauge superfields as

$$\begin{aligned}
\tilde{V}^x &= \frac{M_5^{1/2}}{\sqrt{2}} V^{I=x} + \mathcal{O}\left(M_5^{-5/2}\right), \\
\tilde{\Phi}_S^x &= \frac{M_5^{1/2}}{\sqrt{2}} \Phi_S^{I=x} + \mathcal{O}\left(M_5^{-5/2}\right).
\end{aligned} \tag{3.15}$$

Terms of $\mathcal{O}(M_5^{-5/2})$ are cubic or higher order for the physical fields.

The graviphoton B_μ is identified with $W_\mu^{I=0}$ at the leading order of M_5^{-1}-expansion in our case. Since we are not interested in the fluctuation modes of the gravitational multiplet in this paper, we will replace the graviphoton by its VEV in the following.

3.3 Deviation from BPS limit

5D SUSY transformation is parametrized by an $\mathrm{SU}(2)_{\boldsymbol{U}}$ Majorana spinor ϵ^i $(i = 1, 2)$. This can be rewritten as two 4D Majorana spinors.

$$\begin{aligned}
\epsilon^+ &\equiv \mathcal{P}_\mathrm{R}\epsilon^1 + \mathcal{P}_\mathrm{L}\epsilon^2 , \\
\epsilon^- &\equiv i\left(\mathcal{P}_\mathrm{R}\epsilon^2 + \mathcal{P}_\mathrm{L}\epsilon^1\right) ,
\end{aligned} \tag{3.16}$$

where the projection operators $\mathcal{P}_{\mathrm{R,L}}$ are defined in eq. (A.6). The signs in the superscripts denote the corresponding parity under the orbifold transformation. Thus, after the orbifold projection, 5D $\mathcal{N} = 1$ SUSY is broken to 4D $\mathcal{N} = 1$ SUSY parametrized by ϵ^+. In the following, we will focus on this SUSY and its breaking.

Now, let us define the following quantities.

$$\begin{aligned}
f_G &\equiv \langle e_y{}^4\rangle^{-1}\dot{\sigma} - \frac{2}{3M_5^3}\langle \mathcal{N}_I Y^{I(3)}\rangle , \\
f_\mathrm{h}^\alpha &\equiv \langle e_y{}^4\rangle^{-1}\left\langle \partial_y \mathcal{A}_1^\alpha + i\tilde{\mathcal{F}}_1^\alpha - \left(W_y^I - i\langle e_y{}^4\rangle M^I\right)(gt_I)^\alpha{}_\beta \mathcal{A}_1^\beta + \frac{3}{2}\dot{\sigma}\mathcal{A}_1^\alpha \right\rangle , \\
D_\mathrm{V}^I &\equiv -\langle e_y{}^4\rangle^{-1}\left\langle \partial_y M^I - 2\langle e_y{}^4\rangle Y^{I(3)} + \dot{\sigma} M^I \right\rangle , \\
f_S^I &\equiv -i\langle Y^{I(1)} + iY^{I(2)}\rangle .
\end{aligned} \tag{3.17}$$

Here, we have assumed that the graviphoton W_y^0 does not have a non-zero VEV. Note that f_h^α, D_V^I and f_S^I are proportional to VEVs of the auxiliary fields in Φ^α, V^I and Φ_S^I in the case of $\langle V_y^{(1)} + iV_y^{(2)}\rangle = 0$. Using these quantities, VEVs of SUSY variations for fermions can be written as

$$
\begin{aligned}
\langle \delta\psi^1_{m\mathrm{R}}\rangle &= -\frac{e^\sigma}{3M_5^3}\mathcal{N}_I\bar{f}^I_S(\gamma_{\underline{m}}\epsilon^+)_\mathrm{R}\,, \qquad \langle \delta\psi^2_{m\mathrm{R}}\rangle = -\frac{i}{2}e^\sigma f_G(\gamma_{\underline{m}}\epsilon^+)_\mathrm{R}\,, \\
\langle \delta\psi^1_{y\mathrm{R}}\rangle &= \frac{\langle {e_y}^4\rangle}{2} f_G\epsilon^+_\mathrm{R}\,, \qquad \langle \delta\psi^2_{y\mathrm{R}}\rangle = -i\left(\langle V_y^{(1)}+iV_y^{(2)}\rangle + \frac{\langle {e_y}^4\rangle}{3M_5^3}\langle\mathcal{N}_I\rangle f^I_S\right)\epsilon^+_\mathrm{R}\,, \\
\langle \delta\zeta^\alpha_\mathrm{R}\rangle &= i\left(f^\alpha_\mathrm{h} - \frac{3}{2}f_G\langle\mathcal{A}^\alpha_1\rangle + \frac{i}{M_5^3}f^I_S\langle\mathcal{N}_I\mathcal{A}^\alpha_2\rangle\right)\epsilon^+_\mathrm{R}, \\
\langle \delta\Omega^{I1}_\mathrm{R}\rangle &= \frac{i}{2}\left(D^I_\mathrm{V} + \langle M^I\rangle f_G\right)\epsilon^+_\mathrm{R}\,, \qquad \langle \delta\Omega^{I2}_\mathrm{R}\rangle = \left(-f^I_S + \frac{1}{3M_5^3}\langle M^I\mathcal{N}_J\rangle f^J_S\right)\epsilon^+_\mathrm{R}\,, \qquad (3.18)
\end{aligned}
$$

where the transformation parameter ϵ^+ is assumed as

$$
\epsilon^+(y) = e^{\frac{\sigma(y)}{2}}\epsilon^+_0\,, \qquad (\epsilon^+_0\text{: 4D Majorana constant spinor})\,. \qquad (3.19)
$$

Thus, the quantities defined in eq. (3.17) and $\langle V_y^{(1)}+iV_y^{(2)}\rangle$ characterize the deviation from the BPS limit. However, not all of them are independent quantities. In fact, there are some relations among them. By taking the SUSY variations of the gauge fixing condition (3.4) and using eq. (3.18), we will obtain a relation

$$
\langle\mathcal{N}_I\rangle\left(D^I_\mathrm{V} + \langle M^I\rangle f_G\right) = 0\,. \qquad (3.20)
$$

Similarly, from the gauge fixing condition (3.5), we will obtain

$$
\begin{aligned}
f^1_\mathrm{h} &= \langle\mathcal{A}^2_2\rangle^{-1}\left\{\frac{3}{2}M_5^3 f_G - \sum_{\underline{\alpha},\underline{\beta}=3}^{2n_\mathrm{H}+2}\rho_{\underline{\alpha}\underline{\beta}}\langle\mathcal{A}^{\underline{\alpha}}_2\rangle f^{\underline{\beta}}_\mathrm{h}\right\}, \\
f^2_\mathrm{h} &= -\langle\mathcal{A}^2_2\rangle^{-1}\left\{\sum_{\underline{\alpha}=3}^{2n_\mathrm{H}+2}\langle\bar{\mathcal{A}}^{\underline{\alpha}}_2\rangle f^{\underline{\alpha}}_\mathrm{h} + i\langle\mathcal{N}_I\rangle f^I_S\right\}, \qquad (3.21)
\end{aligned}
$$

where

$$
\langle\mathcal{A}^2_2\rangle = \left(M_5^3 + \left|\langle\mathcal{A}^3_2\rangle\right|^2 + \left|\langle\mathcal{A}^4_2\rangle\right|^2\right)^{1/2}\,. \qquad (3.22)
$$

4. Action after gauge fixing

In this section, we will derive the expression after imposing the gauge fixing conditions (3.1)–(3.5) on the invariant action (2.16).

4.1 Brane action

Before proceeding to the derivation of the bulk action, let us discuss the brane action briefly. In addition to the bulk action, we can introduce the brane actions localized at the fixed boundaries of the orbifold. If we neglect fluctuations of the gravitational fields, the

4D superconformal invariant action [22] can be expressed as follows.

$$\begin{aligned} S_{\rm brane} &= \sum_{\hat{y}=0,\pi R} \int d^5x\, c_{\hat{y}} \delta(y-\hat{y}) \mathcal{L}^{(\hat{y})}_{\rm brane}\,, \\ \mathcal{L}^{(\hat{y})}_{\rm brane} &= \left\{ \int d^2\theta\ f_{\bar{I}\bar{J}}(S) \mathcal{W}^{\bar{I}} \mathcal{W}^{\bar{J}} + \text{h.c.} \right\} - \\ &\quad - e^{2\sigma} \int d^4\theta\ \bar{\Sigma}_{\hat{y}} \Sigma_{\hat{y}} e^{-K(S,\bar{S})} + e^{3\sigma} \left\{ \int d^2\theta\ \Sigma^3_{\hat{y}} P(S) + \text{h.c.} \right\}\,, \end{aligned} \tag{4.1}$$

where $\hat{y} = 0, \pi R$ denote the coordinates of the fixed boundaries, and $c_{\hat{y}}$ are some dimensionless constants which are assumed as small numbers. $f_{\bar{I}\bar{J}}$, K and P are the gauge kinetic functions, the Kähler potential, and the superpotential, respectively. The indices $\bar{I}, \bar{J}$ run over not only the brane localized vector multiplets but also induced ones on the boundaries from the bulk multiplets. The chiral superfield $\Sigma_{\hat{y}} = \varphi^0_{\hat{y}} - \theta\chi^0_{\hat{y}} - \theta^2\mathcal{F}^0_{\hat{y}}$ is a 4D compensator superfield, and S^a are chiral matter superfields. The Weyl weights of the lowest components in $\Sigma_{\hat{y}}$ and S^a are one and zero, respectively. The warp factors in $\mathcal{L}^{(\hat{y})}_{\rm brane}$ come from the induced metric on the boundaries. Note that the 4D compensator $\Sigma_{\hat{y}}$ must be induced from the 5D compensator because the gravity is unique. Since the 5D compensator scalar $\mathcal{A}^{\alpha=2}_2$ has the Weyl weight 3/2, we should identify $\Sigma_{\hat{y}}$ as[10]

$$\Sigma_{\hat{y}} = \Sigma|_{y=\hat{y}}\,, \tag{4.2}$$

where

$$\Sigma = (\Phi^{\alpha=2})^{2/3}\,. \tag{4.3}$$

Chiral matter fields S^a appearing in $S_{\rm brane}$ can contain the induced multiplets on the boundary from the 5D bulk multiplets. However, since the Weyl weight of S^a must be zero, the physical bulk fields Φ^α ($\alpha \geq 3$) can appear in $S_{\rm brane}$ only in the form of

$$S^v_{\hat{y}} \equiv M_5^{-1/2} H^v|_{y=\hat{y}} = \varphi^v_{\hat{y}} - \theta\chi^v_{\hat{y}} - \theta^2\mathcal{F}^v_{\hat{y}}\,, \tag{4.4}$$

where $v = 1, 2, \ldots, n_{\rm H}$, and 5D superfields H^v are defined by

$$H^v \equiv \frac{\sqrt{2}M_5^{3/2}\Phi^{\alpha=2v+2}}{\Phi^{\alpha=2}} = h^v - \theta\chi^v_H - \theta^2\mathcal{F}^v_H\,. \tag{4.5}$$

The numerical factor is for the canonical normalization in the bulk action. Similarly, we will define another bulk fields H^{Cv} ($v = 1, 2, \ldots, n_{\rm H}$) as

$$H^{Cv} \equiv \frac{\sqrt{2}M_5^{3/2}\Phi^{\alpha=2v+1}}{\Phi^{\alpha=2}} = h^{Cv} - \theta\chi^{Cv}_H - \theta^2\mathcal{F}^{Cv}_H\,. \tag{4.6}$$

Note that H^{Cv} are odd under the orbifold parity and vanish on the boundaries, while H^v are even. (See table 1.)

[10] We cannot use $\Phi^{\alpha=1}$ because it is odd under the orbifold parity and vanishes on the boundaries.

The gauge fixing conditions (3.2) and (3.5) are rewritten on the boundary in terms of the components of $\Sigma_{\hat{y}}$ and $S^v_{\hat{y}}$ as

$$\begin{aligned}
\varphi^0_{\hat{y}} &= M_5\left(1+\frac{1}{6M_5^2}\sum_{v=1}^{n_{\rm H}}(|\varphi^v_{\hat{y}}|^2+|\varphi^{Cv}_{\hat{y}}|^2)+\mathcal{O}(M_5^{-4})\right)\,,\\
\chi^0_{\hat{y}} &= \frac{1}{3M_5^2}\varphi^0_{\hat{y}}\sum_{v=1}^{n_{\rm H}}(\bar{\varphi}^v_{\hat{y}}\chi^v_{\hat{y}}+\bar{\varphi}^{Cv}_{\hat{y}}\chi^{Cv}_{\hat{y}})+\mathcal{O}(M_5^{-4})\,,
\end{aligned}\tag{4.7}$$

where $\mathcal{O}(M_5^{-4})$ terms are quartic and higher order terms for the physical fields. These conditions coincide with the gauge fixing conditions in the superconformal formulation of 4D supergravity up to the quadratic order for the physical fields, except for the overall factor of $\varphi^0_{\hat{y}}$. (See the appendix of ref. [11].) The discrepancy of the overall factor leads to a wrong coefficient of the Einstein-Hilbert term. However, this is not a serious problem. Strictly speaking, when we add the brane actions to the bulk action, the $\boldsymbol{D}$-gauge fixing condition (3.1) and (3.2) should be modified so that the 4D Einstein-Hilbert term is canonically normalized after integrating out y-coordinate. However, this modification is almost negligible because of the suppression by the large volume of the extra dimension and the smallness of $c_{\hat{y}}$ in our case. The coefficient of the 4D Einstein-Hilbert term is determined mainly by the bulk term. Thus, the gauge fixing conditions (3.1)–(3.5) can also be applied for the bulk-brane system.

4.2 Bulk action

Now we will derive the bulk action $S_{\rm bulk}$ after the gauge fixing. For simplicity, we will assume that the compensator multiplet and the hidden sector fields Φ^α ($\alpha=1,\ldots,4$) are charged under only the graviphoton W^0_μ. All the directions of the gauging are chosen to σ_3-direction since the gauging along the other directions mixes $\Phi^{2\hat{\alpha}+1}$ and $\Phi^{2\hat{\alpha}+2}$, which have opposite parity eigenvalues. Namely,

$$gt_0=-i\begin{pmatrix} g^0_{\rm c} & & \\ & g^0_{\rm h} & \\ & & \boldsymbol{g}^0\end{pmatrix}\otimes\sigma_3\,,\qquad gt_I=-i\begin{pmatrix} 0 & & \\ & 0 & \\ & & \boldsymbol{g}^I\end{pmatrix}\otimes\sigma_3\,,\qquad (I\neq 0)\tag{4.8}$$

where $\boldsymbol{g}^I\equiv{\rm diag}(g^I_2,g^I_3,\ldots,g^I_{n_{\rm H}})$ ($I=0,1,\ldots,n_{\rm V}$) are $(n_{\rm H}-1)\times(n_{\rm H}-1)$ matrices of the gauge couplings for the hypermultiplets in the visible sector. Note that the gauge couplings $(g^0_{\rm c},g^0_{\rm h},\boldsymbol{g}^0)$ are odd under the orbifold parity since $\Pi_{I=0}=-1$ in table 1. Namely, $(g^0_{\rm c},g^0_{\rm h},\boldsymbol{g}^0)$ have kink profiles.[11] The other gauge couplings are even under the parity and constant in the whole range of y.

Since we are interested only in the visible sector, we will neglect the fluctuation modes of the stabilizer hypermultiplet $(\Phi^{\alpha=3},\Phi^{\alpha=4})$ around the background. Then, the bulk

[11] The kink-type gauge couplings can be realized in SUGRA context by the mechanism proposed in ref. [23].

action is written as

$$
\begin{aligned}
S_{\rm bulk} &= \int d^5x\, \mathcal{L}_{\rm SF} + \mathcal{L}_{\rm SB}\,, \\
\mathcal{L}_{\rm SF} &= \left\{ \int d^2\theta\, \frac{1}{4} T \tilde{\mathcal{W}}^x \tilde{\mathcal{W}}^x + \text{h.c.} \right\} + \frac{e^{2\sigma}}{2} \int d^4\theta\, \frac{T+\bar{T}}{V_T^2} \left(\partial_y \tilde{V}^x + i\tilde{\Phi}_S^x - i\bar{\tilde{\Phi}}_S^x \right)^2 - \\
&\quad - e^{2\sigma} \int d^4\theta\, V_T (\bar{\Sigma}\Sigma)^{3/2} \left\{ 2 - \frac{1}{M_5^3} \left(\bar{H} e^{2\tilde{\boldsymbol{g}}^x \tilde{V}^x} H + \bar{H}^C e^{-2\tilde{\boldsymbol{g}}^x \tilde{V}^x} H^C \right) \right\} + \\
&\quad + e^{3\sigma} \left\{ \frac{1}{M_5^3} \int d^2\theta\, \Sigma^3 H^C \left(\frac{1}{2} \overleftrightarrow{\partial}_y + m_0 T - 2i \tilde{\boldsymbol{g}}^x \tilde{\Phi}_S^x \right) H + \text{h.c.} \right\}, \\
\mathcal{L}_{\rm SB} &= -e^{4\sigma} \langle e_y{}^4 \rangle \left\{ \hat{O}_h (|h|^2 + |h^C|^2) + f_G \left(1 + \frac{|\mathcal{A}_2^3|^2 + |\mathcal{A}_2^4|^2}{M_5^3} \right) (\bar{h} m_0 h - \bar{h}^C m_0 h^C) \right\} - \\
&\quad - e^{4\sigma} \langle e_y{}^4 \rangle \hat{O}_\phi \left((\phi^x)^2 \right) - e^{2\sigma} f_G \left\{ \langle e_y{}^4 \rangle \phi^x \tilde{D}^x - \frac{1}{2} (\tilde{\chi}_S^x \tilde{\lambda}^x + \text{h.c.}) \right\} + \cdots\,, \qquad (4.9)
\end{aligned}
$$

where H^v, H^{Cv}, and Σ are defined in the previous subsection. The index $v = 2, 3, \ldots, n_{\rm H}$ for the hypermultiplets is suppressed, and summations for the indices $x = 1, 2, \ldots, n_{\rm V}$ are implicit. The derivative operator $\overleftrightarrow{\partial}_y$ is defined as $A \overleftrightarrow{\partial}_y B \equiv A\partial_y B - B\partial_y A$. The mass matrix m_0 and the physical (dimensionful) gauge couplings $\tilde{\boldsymbol{g}}^x$ are defined as

$$
m_0 \equiv M_5 \boldsymbol{g}^0\,, \qquad \tilde{\boldsymbol{g}}^x \equiv \frac{\sqrt{2}}{M_5^{1/2}} \boldsymbol{g}^{I=x}\,, \qquad (4.10)
$$

and the superfield T and the operators $\hat{O}_h$ and $\hat{O}_\phi$ are

$$
\begin{aligned}
T &\equiv -\frac{2i}{M_5} \Phi_S^0\,, \\
\hat{O}_h &\equiv \left(1 + \frac{|\mathcal{A}_2^3|^2 + |\mathcal{A}_2^4|^2}{M_5^3} \right) \left\{ g_{\rm c}^0 D_{\rm V}^0 - \frac{f_{\rm h}^1}{M_5^{3/2}} \langle e_y{}^4 \rangle^{-1} \left(\partial_y + \langle e_y{}^4 \rangle M_5 g_{\rm c}^0 + \frac{3}{2}\dot{\sigma} \right) \right\} - \\
&\quad - \frac{2 f_{\rm h}^1}{M_5^{9/2}} \langle e_y{}^4 \rangle^{-1} \text{Re}(\bar{\mathcal{A}}_2^3 \partial_y \mathcal{A}_2^3 + \bar{\mathcal{A}}_2^4 \partial_y \mathcal{A}_2^4), \qquad (4.11) \\
\hat{O}_\phi &\equiv \left\{ -\frac{f_G}{2} \langle e_y{}^4 \rangle^{-1} (\partial_y + 2\dot{\sigma}) + \frac{f_G^2}{2} + \frac{2}{M_5^2} |f_S^0|^2 - \frac{4 g_{\rm h}^0}{3 M_5^2} \left(\mathcal{A}_2^3 f_{\rm h}^4 - \mathcal{A}_2^4 f_{\rm h}^3 \right) - \frac{4 g_{\rm c}^0}{3 M_5^{1/2}} f_{\rm h}^1 \right\}.
\end{aligned}
$$

Here, we have used the relation

$$
D_{\rm V}^0 = -M_5 f_G\,, \qquad (4.12)
$$

which is derived from eq. (3.20) under the assumption that $\langle \phi^x \rangle = 0$.[12] The ellipsis in eq. (4.9) denotes quartic or higher order terms for the physical fields. Such terms are suppressed by the large 5D Planck mass M_5 and are negligible. Field independent terms are dropped here.

In the above expression, fermionic components and fluctuation modes of auxiliary fields in Σ and T only contribute $\mathcal{O}(\kappa)$ quartic or higher order terms that are neglected here.

[12]In this paper, we assume that only $\mathcal{A}_2^{\alpha=3,4}$ have nontrivial vacuum configurations.

Thus, in eq. (4.9), Σ and T can be understood as the following spurion superfields.

$$\begin{aligned}\Sigma &= \left(M_5 + \frac{\left|\langle\mathcal{A}_2^3\rangle\right|^2 + \left|\langle\mathcal{A}_2^4\rangle\right|^2}{3M_5^2}\right) - \theta^2\mathcal{F}_\Sigma\,,\\ T &= \left(\langle e_y{}^4\rangle - \frac{2i}{M_5}\langle W_y^0\rangle\right) - \theta^2\mathcal{F}_T\,,\end{aligned} \tag{4.13}$$

where $\mathcal{F}_T \equiv -2ie^\sigma\left(\langle V_y^{(1)} + iV_y^{(2)}\rangle + \langle e_y{}^4\rangle M_5^{-1} f_S^0\right)$. From eqs. (2.8) and (4.13), we can see that T and V_T correspond to the radion superfield. We will discuss it in the next section. All the SUSY breaking terms in $\mathcal{L}_{\rm SB}$ involve quantities defined in eq. (3.17) and thus vanish in the case of a BPS background.

4.3 Radion superfield

Here, we will consider the case that the background is BPS. In this case, $\mathcal{L}_{\rm SB}$ in eq. (4.9) vanishes and the action can be expressed by only superfields. Besides, V_T can be expressed by T as[13]

$$V_T = \frac{T+\bar{T}}{2}\,, \tag{4.14}$$

where $\mathcal{F}_T = -2ie^\sigma\langle V_y^{(1)} + iV_y^{(2)}\rangle$ in T. The above relation always holds only if $f_S^0 = 0$, i.e., only one of $\mathcal{A}_2^{\alpha=3}$ and $\mathcal{A}_2^{\alpha=4}$ has a nonzero vacuum configuration. (See eqs. (3.17) and (B.3).) Here, we will rescale the compensator superfield as

$$\Sigma' \equiv \frac{\Sigma}{M_5} = \left(1 + \frac{\left|\langle\mathcal{A}_2^3\rangle\right|^2 + \left|\langle\mathcal{A}_2^4\rangle\right|^2}{3M_5^3}\right) - \theta^2\mathcal{F}'_\Sigma\,. \tag{4.15}$$

Then, the superfield lagrangian $\mathcal{L}_{\rm SF}$ becomes

$$\begin{aligned}\mathcal{L}_{\rm SF} = &\left\{\int d^2\theta\,\frac{1}{4}T\tilde{\mathcal{W}}^x\tilde{\mathcal{W}}^x + \text{h.c.}\right\} + e^{2\sigma}\int d^4\theta\,\frac{2}{T+\bar{T}}\left(\partial_y\tilde{V}^x + i\tilde{\Phi}_S^x - i\bar{\tilde{\Phi}}_S^x\right)^2 - \\ &- e^{2\sigma}\int d^4\theta\,\frac{T+\bar{T}}{2}(\bar{\Sigma}'\Sigma')^{3/2}\left\{2M_5^3 - \bar{H}e^{2\tilde{\boldsymbol{g}}^x\tilde{V}^x}H - \bar{H}^Ce^{-2\tilde{\boldsymbol{g}}^x\tilde{V}^x}H^C\right\} + \\ &+ e^{3\sigma}\left\{\int d^2\theta\,\Sigma'^3 H^C\left(\frac{1}{2}\overleftrightarrow{\partial}_y + m_0T - 2i\tilde{\boldsymbol{g}}^x\tilde{\Phi}_S^x\right)H + \text{h.c.}\right\}\,.\end{aligned} \tag{4.16}$$

In the limit that $\Lambda_{\rm hid}/M_5 \to 0$, i.e., $\langle\mathcal{A}_2^{\alpha=3,4}\rangle/M_5^{3/2} \to 0$, the warp factor is calculated as

$$\sigma(y) = -\frac{2\langle e_y{}^4\rangle M_5 g_{\rm c}^0}{3}y\,, \tag{4.17}$$

by solving the Killing spinor equation. Here, the normalization of the warp factor is chosen as $\sigma(0) = 0$. This is the case of the supersymmetric Randall-Sundrum model.[14] In this case, the above result becomes very similar to the result of ref. [16], except for the dependence of the *radion superfield* T in the warp factor. We can reproduce the T-dependence in the

[13] This relation is also mentioned in ref. [20].

[14] Note that the gauge coupling $g_{\rm c}^0$ is odd under the orbifold parity.

warp factor of ref. [16] by the following redefinition.

$$\begin{aligned} V_T &\to \exp\left\{-\left(\frac{T+\bar{T}}{2\langle e_y{}^4\rangle}-1\right)\sigma\right\} V_T\,, \\ \Sigma' &\to \exp\left\{\left(\frac{T}{\langle e_y{}^4\rangle}-1\right)\sigma\right\}\Sigma'\,. \end{aligned} \tag{4.18}$$

Note that this redefinition does not change the lowest components of V_T and Σ', but change only their auxiliary fields.[15] In this case, however, the relation (4.14) no longer holds, and we cannot reproduce the T-dependence of ref. [16] in the parts of $d^4\theta$-integration in the action.

The reason for the discrepancy in the T-dependence between our action and that of ref. [16] is as follows. As we can see from eq. (4.17), the warp factor e^σ has an $\langle e_y{}^4\rangle$-dependence. In ref. [16], all $\langle e_y{}^4\rangle$ appearing in the action are promoted to the radion superfield T. However, note that the $\langle e_y{}^4\rangle$-dependence of the warp factor is induced by solving the equation of motion, or the Killing spinor equation in the BPS case. In other words, $\sigma(y)$ is independent of $\langle e_y{}^4\rangle$ at the original SUGRA action. ($e_{\underline{m}}^{\;\underline{n}}$ is independent of $e_y{}^4$.) All $\langle e_y{}^4\rangle$ except for those in the warp factor should be incorporated into the superfields because they originate from the fünfbein in the superconformal invariant action. On the other hand, $\langle e_y{}^4\rangle$ in the warp factor appears only after substituting the background solution into the action. Therefore, this $\langle e_y{}^4\rangle$ should not be accompanied by the auxiliary field $\mathcal{F}_T$.

4.4 Comment on Scherk-Schwarz breaking

Next, we will briefly comment on the relation between nonzero $\mathcal{F}_T$ and Scherk-Schwarz (SS) SUSY breaking [24] for further understanding of the radion superfield. In the framework of superconformal gravity, twisting boundary conditions for fields with $\mathrm{SU}(2)_R$ indices can be realized by twisting only the compensator gauge fixing as

$$\mathcal{A}_i^a = \left(e^{i\alpha(y)\vec{\omega}\cdot\vec{\sigma}}\right)^a{}_i\left(M_5^3+\sum_{\underline{\alpha}=3}^{2n_{\rm H}}\left|\mathcal{A}_2^{\underline{\alpha}}\right|^2\right)^{1/2}, \qquad (a=1,2)\,, \tag{4.19}$$

where $\vec{\omega}=(\omega^1,\omega^2,\omega^3)$ is a unit twist vector, while the physical fields are kept untwisted. In fact, we can move to the usual SS twisting for the physical fields by rotating back the above gauge fixing to the usual one (3.3) with an $\mathrm{SU}(2)_{\boldsymbol{U}}$ matrix. For the single-valuedness of the compensator fields, the twisting parameter $\alpha(y)$ satisfies

$$\alpha(y+2\pi R)=\alpha(y)+2n\pi\,, \qquad n\text{: nonzero integer})\,. \tag{4.20}$$

Notice that we can also move to the untwisted compensator by $\mathrm{SU}(2)_C$ rotation mentioned in Sect.3.1,

$$\mathcal{A}_i^a \to U^a{}_b\mathcal{A}_i^b\,, \qquad \zeta^a \to U^a{}_b\zeta^b\,, \tag{4.21}$$

[15] We have assumed that $\langle W_y^0\rangle=0$.

where $U^a{}_b$ $(a, b = 1, 2)$ is an $\mathrm{SU}(2)_C$ matrix defined as

$$U^a{}_b \equiv \left(e^{-i\alpha(y)\vec{\omega}\cdot\vec{\sigma}}\right)^a{}_b . \tag{4.22}$$

From eq. (B.3), we can see that the $\mathrm{SU}(2)_{\boldsymbol{U}}$ gauge fields $V_y^{(r)}$ $(r = 1, 2, 3)$ are shifted in this basis as

$$V_y^{(r)} \to V_y^{(r)} + \frac{\partial_y \alpha \left|\mathcal{A}_2^2\right|^2}{M_5^3}\omega^r = V_y^{(r)} + \partial_y \alpha \left(1 + \frac{1}{M_5^3}\sum_{\underline{\alpha}=3}^{2n_{\mathrm{H}}} \left|\mathcal{A}_2^{\underline{\alpha}}\right|^2\right)\omega^r , \tag{4.23}$$

where we have used the gauge fixing conditions in the second equality. Thus, the SS twisting induces the non-vanishing Wilson line of the $\mathrm{SU}(2)_{\boldsymbol{U}}$ gauge field.

$$\int_0^{2\pi R} dy\, \langle V_y^{(r)}\rangle = 2\pi n(1+\delta)\omega^r , \tag{4.24}$$

where

$$\delta \equiv \int_0^{2\pi R} dy\, \frac{\partial_y \alpha(y)}{2\pi n M_5^3}\left(\left|\langle\mathcal{A}_2^3\rangle(y)\right|^2 + \left|\langle\mathcal{A}_2^4\rangle(y)\right|^2\right) . \tag{4.25}$$

We have used eq. (4.20) and assumed that the SS twisting is the only source of the non-vanishing Wilson line. In fact, the stabilizer fields $\mathcal{A}_2^{\alpha=3,4}$ can also contribute to the Wilson line, but it is estimated as $\mathcal{O}(\Lambda_{\mathrm{hid}}^3/M_5^3)$ from eq. (B.3) and negligible.

Thus, the SS twisting is equivalent to the non-vanishing Wilson line. Here, notice that the consistency of the SS twisting with the orbifold transformation characterized by table 1 requires $\omega^3 = 0$.[16] Thus, we can ignore the dependence of $V_y^{(3)}$ in the action, as we did. As mentioned in the previous subsection, the auxiliary field of radion is provided by $\mathcal{F}_T = -2ie^{\sigma}\langle V_y^{(1)} + iV_y^{(2)}\rangle$. Therefore, we can conclude that the SS SUSY breaking is equivalent to the breaking due to a nonzero $\mathcal{F}_T$, as is well-known in the case of flat spacetime [16].

According to ref. [25], however, the twisted boundary condition leads to inconsistent theory in the slice of AdS spacetime. In the superconformal formulation, this can be seen from the appearance of a nonzero mass of the graviphoton B_μ, which is proportional to $\vec{\omega}$ and g_{c}^0. Thus, this inconsistency of the theory can be avoided if $g_{\mathrm{c}}^0 = 0$ and the warp factor can be generated only from the vacuum configuration of the bulk scalars.

4.5 Geometry mediated SUSY breaking

Finally, we will discuss the SUSY breaking effect induced by the non-BPS background geometry. In order to focus on this effect, we will consider the case that there is no SS twisting discussed in the previous subsection. In this case, SUSY breaking is characterized by the quantities defined in eq. (3.17). Among them, only $f_{\mathrm{h}}^{3,4}$ and f_G are independent in our case. In fact, $f_{\mathrm{h}}^{1,2}$ and D_{V}^0 are related to them through eqs. (3.21) and (4.12),

[16] In general, the orbifold transformation can mix two components of an $\mathrm{SU}(2)_{\boldsymbol{U}}$-doublet spinor ψ^i as $\psi^i(-y) = \Pi\gamma_5 M^i{}_j\psi^j(y)$, where the mixing matrix $M^i{}_j$ satisfies $M^* = -\sigma_2 M\sigma_2$. We have chosen as $M = \sigma_3$ in this paper.

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and the others are zero. Here, we assume that only one complex scalar $\mathcal{A}_2^3$ or $\mathcal{A}_2^4$ has a nontrivial field configuration so that $f_S^0 = 0$.[17] The quantities $f_{\rm h}^{3,4}$ and f_G characterize SUSY breaking in the stabilization sector and the deviation of the geometry from the BPS limit, respectively. They are calculated immediately once the non-BPS background is found.

The action is calculated as

$$\begin{aligned}
S &= \int d^5x\, \mathcal{L}_{\rm SF} + \mathcal{L}_{SB}\,, \\
\mathcal{L}_{\rm SF} &= \left\{ \int d^2\theta\, \frac{1}{4}\tilde{\mathcal{W}}^x\tilde{\mathcal{W}}^x + {\rm h.c.} \right\} + e^{2\sigma}\int d^4\theta\, \left(\partial_y \tilde{V}^x + i\tilde{\Phi}_S^x - i\bar{\tilde{\Phi}}_S^x \right)^2 + \\
&\quad + e^{2\sigma}\int d^4\theta\, \left(\bar{H} e^{2\tilde{\boldsymbol{g}}^x\tilde{V}^x} H + \bar{H}^C e^{-2\tilde{\boldsymbol{g}}^x\tilde{V}^x} H^C \right) + \\
&\quad + e^{3\sigma}\left\{ \int d^2\theta\, H^C \left(\frac{1}{2}\overleftrightarrow{\partial}_y + m_0 - 2i\tilde{\boldsymbol{g}}^x\tilde{\Phi}_S^x \right) H + {\rm h.c.} \right\}\,, \\
\mathcal{L}_{\rm SB} &= e^{4\sigma} f_G \Bigg\{ (\partial_y + 3\dot{\sigma} + 2M_5 g_{\rm c}^0 - f_G)(\phi^x)^2 + \\
&\qquad + \left(\frac{3}{2}\partial_y + \frac{5}{2}M_5 g_{\rm c}^0 + \frac{9}{4}\dot{\sigma} \right)(|h|^2 + |h^C|^2) - (\bar{h}m_0 h - \bar{h}^C m_0 h^C) + \\
&\qquad + \frac{e^{-2\sigma}}{2}(\tilde{\chi}_S^x\tilde{\lambda}^x + {\rm h.c.}) + \phi^x(\bar{h}\tilde{\boldsymbol{g}}^x h - \bar{h}^C\tilde{\boldsymbol{g}}^x h^C) \Bigg\} + \cdots\,,
\end{aligned} \tag{4.26}$$

where the ellipsis denotes the negligible quartic or higher order terms for the physical fields. Here, we have rescaled y so that $\langle e_y{}^4 \rangle = 1$, and used

$$f_{\rm h}^1 \simeq \frac{3}{2}M_5^{3/2} f_G\,, \tag{4.27}$$

which follows from eq. (3.21) under the assumption $\Lambda_{\rm hid} \ll M_5$. We have redefine the auxiliary field $\tilde{D}^x$ in $\tilde{V}^x$ as $\tilde{D}^x \to \tilde{D}^x - 2e^{2\sigma} f_G \phi^x$ in order to absorb the explicit dependence of $\tilde{D}^x$ in $\mathcal{L}_{\rm SB}$ of eq. (4.9).

Note that all SUSY breaking terms are proportional to f_G, which characterizes the deviation of the warp factor $\sigma(y)$ from the BPS limit. This means that SUSY breaking in the visible sector mainly comes from only the deformation of the background geometry. We can also see that no A-terms or B-terms for the matter hyperscalars are induced. Since the physical gauge scalar superfields $\tilde{\Phi}_S^x$ ($x = 1, 2, \ldots, n_{\rm V}$) are odd under the orbifold transformation, they have no zero-modes and decouple below the compactification scale R^{-1}.[18] Thus, the only SUSY breaking terms induced by the deformed geometry are the mass terms for the hyperscalars in the low-energy effective theory.

We can check that the non-BPS bulk geometry does not cause SUSY breaking in the brane actions at tree level. This can easily be understood from the fact that $f_G(y)$ is an odd function under the orbifold parity and vanishes on the boundaries.

[17] One example of such non-BPS configuration is found in ref. [26] in the 5D global SUSY model. The authors insist that their solution can be embedded into 5D SUGRA.

[18] Since $\Lambda_{\rm hid}^{-1}$ is the characteristic length of the nontrivial field configuration, the radius R is generally larger than $\Lambda_{\rm hid}^{-1}$.

5. Summary and comments

We have proposed a practical method of deriving 5D SUGRA action on a general warped geometry using 4D superfields. Our method is based on the superconformal formulation of 5D SUGRA discussed in refs. [12]–[15]. We have expressed an invariant action in 4D $\mathcal{N}=1$ superspace in the case that fluctuation modes of the gravitational fields are neglected. This greatly simplifies calculations and makes the procedure transparent thanks to the well-known 4D superfield formalism. A significant advantage of our method is that we can deal with a *general warped geometry* including a non-BPS background in the superfield formalism.

In the case of a non-BPS background, SUSY breaking terms appear after the superconformal gauge fixing. This might seem an explicit breaking of SUSY by the gauge fixing conditions. However, this is not the case. Notice that the genuine SUSY transformation $\tilde{\delta}_Q$ is different from a fermionic transformation $\tilde{\delta}'_Q$ generated by acting a differential operator $\epsilon_{\rm R}^+\hat{Q}+\bar{\epsilon}_{\rm R}^+\bar{\hat{Q}}$ on each superfield, where

$$\hat{Q}_\alpha=\frac{\partial}{\partial\theta^\alpha}-i\left(\sigma^{\underline{m}}\bar{\theta}\right)_\alpha\partial_m\,,\qquad \bar{\hat{Q}}^{\dot{\alpha}}=\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}-i\left(\bar{\sigma}^{\underline{m}}\theta\right)^{\dot{\alpha}}\partial_m\,. \tag{5.1}$$

The genuine SUSY $\tilde{\delta}_Q$ is defined as a combination of Q-transformation δ_Q, the conformal supersymmetry δ_S, and the special conformal boost δ_K,

$$\tilde{\delta}_Q(\epsilon)\equiv\delta_Q(\epsilon)+\delta_S(\eta(\epsilon))+\delta_K(\xi_K(\epsilon))\,, \tag{5.2}$$

where transformation parameters η^i and $\xi_K^{\underline{\mu}}$ are determined so that $\tilde{\delta}_Q$ preserves the superconformal gauge fixing conditions. (See eq. (D.6) in the second paper of ref. [12] for the explicit expressions of η^i and $\xi_K^{\underline{\mu}}$.) On the other hand, we have defined $\tilde{\delta}'_Q$ so that it preserves the gravitational background (2.7), i.e.,[19]

$$\eta^i=\frac{i}{2}\dot{\sigma}\gamma_5\epsilon^i\,,\qquad \xi_K^{\underline{\mu}}=0\,. \tag{5.3}$$

Namely, the gauge fixing conditions break $\tilde{\delta}'_Q$-symmetry, not the genuine SUSY $\tilde{\delta}_Q$. The latter is broken by the non-BPS background, and thus this breaking is spontaneous. Notice that the difference between $\tilde{\delta}_Q$ and $\tilde{\delta}'_Q$ is a choice of η and ξ_K, and the superconformal transformations of $\mathcal{A}_i^\alpha$, W_μ^I and M^I include neither η nor ξ_K. Therefore, both transformations $\tilde{\delta}_Q$ and $\tilde{\delta}'_Q$ are identical for these fields. Since the superpartners of these fields are defined by $\tilde{\delta}_Q\mathcal{A}_i^\alpha$, $\tilde{\delta}_QW_\mu^I$ and $\tilde{\delta}_QM^I$, the superfields constructed by $\tilde{\delta}'_Q$ describe the correct supermultiplets for *the on-shell fields*. All the transformations including η or ξ_K involve the auxiliary fields. This means that the deviation of $\tilde{\delta}'_Q$ from $\tilde{\delta}_Q$ can be pushed into the definition of the auxiliary fields of the superfields. In fact, $\dot{\sigma}$ in eqs. (2.10) and (2.14) corresponds to the contributions of η in eq. (5.3).

Our result can be compared with that of ref. [16] in the case of a BPS background. We saw that the radion superfield originates from two different kinds of superconformal multiplets in ref. [14]. One is a 4D real general multiplet whose lowest component is ${e_y}^4$

[19] In order to construct 4D superfields, the transformation parameter ϵ^i is restricted to $\epsilon^+=e^{\frac{\sigma}{2}}\epsilon_0^+$ (ϵ_0^+: 4D Majorana constant spinor).

and the other is a 4D chiral multiplet whose lowest component is $(W_y^0 + ie_y{}^4 M^0)/2$. On the gravitational background (2.7), the corresponding superfield of the former multiplet is a spurion superfield V_T while the latter becomes the gauge scalar superfield Φ_S^0 for the graviphoton multiplet.[20] Although these superfields have different origins, they describe almost the same degrees of freedom on the gravitational background,[21] except for $Y^{0(1)} + iY^{0(2)}$, which has a vanishing VEV if only one scalar of the stabilizer hypermultiplet has a nontrivial configuration. In such a case, both V_T and Φ_S^0 are expressed by a single chiral spurion field T, which is commonly called the *radion superfield*. Since we started from the off-shell SUGRA formulation, we can derive the dependence of T in the action without any ambiguity. Our result is slightly different from that of ref. [16] by the T-dependence in the warp factor. This discrepancy stems from the fact that the $\langle e_y{}^4 \rangle$-dependence of the warp factor is induced by solving the equation of motion (or the Killing spinor equation in the BPS case). In other words, $\sigma(y)$ is independent of $\langle e_y{}^4 \rangle$ at the original SUGRA action because $e_m{}^{\underline{n}}$ is independent of $e_y{}^4$. Therefore, $\langle e_y{}^4 \rangle$ in the warp factor should not be accompanied by the auxiliary field $\mathcal{F}_T$. Using our identification of the radion superfield, we can explicitly see that SUSY breaking by the auxiliary field $\mathcal{F}_T$ of the radion superfield is equivalent to the Scherk-Schwarz breaking, as is well-known in flat spacetime.

We also discussed SUSY breaking induced by a deformed geometry. Here, we have assumed that the deformed (non-BPS) geometry comes from the backreaction of a non-trivial scalar configuration which is relevant to the radius stabilization. In the case that the stabilization sector is decoupled from our visible sector, all SUSY breaking effects are induced through the background geometry. In fact, the dominant SUSY breaking terms are proportional to f_G, which characterizes the deformation of the geometry. (See eq. (4.26).) Namely, f_G is a y-dependent order parameter of SUSY breaking in this case. We can see that the deformed geometry only induces the masses for the bulk scalars besides the scalar trilinear coupling involving the gauge scalars ϕ^x. It should also be noted that the geometry mediated SUSY breaking induces no SUSY breaking effects on the boundaries at tree level because the order parameter $f_G(y)$ is an odd function under the orbifold parity and vanishes on the boundaries.

Finally, we would like to comment on the recently appeared preprint [20] related to our work. In ref. [20], the authors express 5D superconformal gravity action in terms of the action formulae for 4D superconformal gravity. Our action before the superconformal gauge fixing should be obtained by fixing the gravitational background in their action. The radion spurion superfield discussed in our paper is consistent with their identification of the radion multiplet. In our paper, we further show the action after the gauge fixings explicitly which is useful to derive the effective theory even on a non-BPS background (general warped background).

[20] As mentioned in ref. [14], both multiplets have nontrivial transformation properties under the full 5D superconformal transformation, and it is a hard task to find how they appear in 4D action formulae in a 5D superconformal invariant way. On the other hand, our superfield formalism is much easier to deal with because we focus on only the $\tilde{\delta}'_Q$-symmetry, which is just part of the full superconformal symmetry.

[21] The fermionic component of Φ_S^0 only contributes to quartic or higher order terms suppressed by the 5D Planck mass M_5, which are neglected in this paper.

Acknowledgments

The authors would like to thank Ian-Woo Kim and Keisuke Ohashi for useful discussions. H.A. is supported by KRF PBRG 2002-070-C00022. Y.S. is supported from the Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCSEC) funded by the Korea Science and Engineering Foundation and the Korean Ministry of Science.

A. Notation

Basically, we follow the notation of refs. [13, 14]. The metric convention is

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1, -1)\,. \tag{A.1}$$

We choose the following representation for γ-matrices.

$$\gamma^0 = \begin{pmatrix} & -1 \\ -1 & \end{pmatrix}, \qquad \gamma^k = \begin{pmatrix} & \sigma_k \\ -\sigma_k & \end{pmatrix}, \qquad \gamma^4 = \begin{pmatrix} -i & \\ & i \end{pmatrix}, \tag{A.2}$$

where σ_k $(k = 1, 2, 3)$ are the Pauli matrices. These satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\,. \tag{A.3}$$

The $\mathrm{SU}(2)_{\boldsymbol{U}}$ Majorana condition is

$$\bar{\Omega}^{Ii} \equiv (\Omega^I_i)^\dagger \gamma_0 = (\Omega^{Ii})^T C_5\,, \tag{A.4}$$

where the 5D charge conjugation matrix C_5 is

$$C_5 = \begin{pmatrix} -i\sigma_2 & \\ & i\sigma_2 \end{pmatrix}. \tag{A.5}$$

The chirality matrix γ_5 in four dimensions is defined as $\gamma_5 \equiv i\gamma^4$, and the projection operators $\mathcal{P}_{\mathrm{R,L}}$ are defined as

$$\mathcal{P}_{\mathrm{R}} \equiv \frac{1+\gamma_5}{2}\,, \qquad \mathcal{P}_{\mathrm{L}} \equiv \frac{1-\gamma_5}{2}\,. \tag{A.6}$$

In the superfield formalism, we use the 2-component notation for spinors. For a 4-component spinor ψ, a 2-component spinor ψ_{R} is defined as

$$\mathcal{P}_{\mathrm{R}}\psi \equiv \begin{pmatrix} \psi_{\mathrm{R}} \\ 0 \end{pmatrix}. \tag{A.7}$$

Using this notation, we can rewrite the 4-component spinors Ω^{Ii} in terms of 2-component spinors Ω^{Ii}_{R} owing to the $\mathrm{SU}(2)_{\boldsymbol{U}}$ Majorana condition (A.4).

$$\Omega^{I1} = \begin{pmatrix} \Omega^{I1}_{\mathrm{R}} \\ -\bar{\Omega}^{I2}_{\mathrm{R}} \end{pmatrix}, \qquad \Omega^{I2} = \begin{pmatrix} \Omega^{I2}_{\mathrm{R}} \\ \bar{\Omega}^{I1}_{\mathrm{R}} \end{pmatrix}. \tag{A.8}$$

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Similarly, we can rewrite the $Usp(2, 2n_{\mathrm{H}})$ Majorana spinors ζ^α as,

$$\zeta^1 = \begin{pmatrix} \zeta^1_{\mathrm{R}} \\ -\bar{\zeta}^2_{\mathrm{R}} \end{pmatrix}, \qquad \zeta^2 = \begin{pmatrix} \zeta^2_{\mathrm{R}} \\ \bar{\zeta}^1_{\mathrm{R}} \end{pmatrix}, \qquad \zeta^3 = \begin{pmatrix} \zeta^3_{\mathrm{R}} \\ -\bar{\zeta}^4_{\mathrm{R}} \end{pmatrix}, \qquad \zeta^4 = \begin{pmatrix} \zeta^4_{\mathrm{R}} \\ \bar{\zeta}^3_{\mathrm{R}} \end{pmatrix}, \qquad \ldots . \tag{A.9}$$

We take a convention of ref. [27] for the contraction of 2-component spinors.

The indices $\alpha, \beta, \ldots$ and $i, j, \ldots$ are lowered (or raised) by the antisymmetric tensors $\rho_{\alpha\beta}$ ($\rho^{\alpha\beta}$) and ϵ_{ij} (ϵ^{ij}), respectively.[22] Here, $\epsilon_{12} = \epsilon^{12} = 1$ and both $\rho_{\alpha\beta}$ and $\rho^{\alpha\beta}$ have the following form in the standard representation [28].

$$\rho = \epsilon \otimes \mathbf{1}_{n_{\mathrm{H}}+1}\,, \qquad (\epsilon = i\sigma_2) \tag{A.10}$$

B. Invariant action on gravitational background

After performing the integration of θ and substituting eqs. (2.10) and (2.14) into each component of superfield, the superspace expression (2.16) becomes

$$\begin{aligned}
S &= \int d^5x\, \mathcal{L}\,, \\
\mathcal{L} &= e\Bigg[-\frac{\mathcal{N}_{IJ}}{2}\left(-\frac{1}{4}\mathcal{F}^{I\mu\nu}\mathcal{F}^J_{\mu\nu} + \frac{1}{2}\partial^\mu M^I \partial_\mu M^J - Y^{Ii}{}_j Y^{Jj}{}_i + 2i\bar{\Omega}^{Ii}\gamma^\mu \mathcal{D}_\mu \Omega^J_i\right) + \\
&\quad + \mathcal{N}_{IJK}\left(\frac{i}{4}\bar{\Omega}^{Ii}\gamma^{\mu\nu}\Omega^J_i F^K_{\mu\nu} + i\bar{\Omega}^{Ii}Y^J_{ij}\Omega^{Kj}\right)\Bigg] + \frac{1}{8}C_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}W^I_\mu F^J_{\nu\rho}F^K_{\sigma\tau} + \\
&\quad + e\Bigg[\left(1 + \frac{(\langle e_y{}^4\rangle^{-1}W^0_y)^2}{(M^0)^2}\right)\tilde{\mathcal{F}}^\alpha_i d_\alpha{}^\beta \tilde{\mathcal{F}}^i_\beta + \mathcal{D}^\mu \mathcal{A}^\alpha_i d_\alpha{}^\beta \mathcal{D}_\mu \mathcal{A}^i_\beta - 2i\bar{\zeta}^\alpha d_\alpha{}^\beta \gamma^\mu \mathcal{D}_\mu \zeta_\beta + \\
&\qquad + \mathcal{A}^\alpha_i d_\alpha{}^\beta \left(M^I(gt_I)M^J(gt_J)\right)_\beta{}^\gamma \mathcal{A}^i_\gamma - 8i\mathcal{A}^\alpha_i d_\alpha{}^\beta \bar{\Omega}^{Ii}(gt_I)_\beta{}^\gamma \zeta_\gamma + \\
&\qquad + 2\mathcal{A}^\alpha_i d_\alpha{}^\beta Y^{Iij}(gt_I)_\beta{}^\gamma \mathcal{A}_{\gamma j} + 2i\bar{\zeta}^\alpha d_\alpha{}^\beta M^I (gt_I)_\beta{}^\gamma \zeta_\gamma\Bigg] - \\
&\quad - e\left(\frac{1}{8}\mathcal{N} - \frac{3}{16}\mathcal{A}^\alpha_i d_\alpha{}^\beta \mathcal{A}^i_\beta\right)\mathcal{R}\,,
\end{aligned} \tag{B.1}$$

where summations for the indices $I, J, K = 0, 1, \ldots, n_{\mathrm{V}}$; $\alpha, \beta, \gamma = 1, 2, \ldots, 2n_{\mathrm{H}} + 2$, and $i, j = 1, 2$ are implicit, and $\mathcal{N}_{IJ} \equiv \partial^2\mathcal{N}/\partial M^I \partial M^J$, $\mathcal{N}_{IJK} \equiv \partial^3\mathcal{N}/\partial M^I \partial M^J \partial M^K$. Here, we have used the 5D spinor notation. Namely, $\bar{\Omega}^{Ii}$ ($\bar{\zeta}^\alpha$) denote $SU(2)_{\boldsymbol{U}}$ ($Usp(2, 2n_{\mathrm{H}})$) Majorana conjugates of Ω^{Ii} (ζ^α). $\mathcal{R}$ is the background value of 5D Ricci scalar and

$$\begin{aligned}
e &\equiv \det(e_\mu{}^{\underline{\nu}}) = e^{4\sigma}\,, \\
\tilde{\mathcal{F}}^\alpha_i &\equiv \mathcal{F}^\alpha_i - M^0(gt_0)^\alpha{}_\beta \mathcal{A}^\beta_i\,, \\
\mathcal{D}_\mu \mathcal{A}^\alpha_i &\equiv \partial_\mu \mathcal{A}^\alpha_i - V_{\mu ij}\mathcal{A}^{\alpha j} - W^I_\mu (gt_I)^\alpha{}_\beta \mathcal{A}^\beta_i\,, \\
\mathcal{D}_\mu \zeta^\alpha &\equiv \left(\partial_\mu - \frac{1}{4}\omega_\mu{}^{\underline{\nu\rho}}\gamma_{\underline{\nu\rho}}\right)\zeta^\alpha - W^I_\mu (gt_I)^\alpha{}_\beta \zeta^\beta\,, \\
\mathcal{R} &= 4\langle e_y{}^4\rangle^{-2}(2\ddot{\sigma} + 5\dot{\sigma}^2)\,.
\end{aligned} \tag{B.2}$$

[22]We take the northwest-to-southeast contraction convention for these indices.

Eq. (B.1) coincides with the invariant action in refs. [13, 15] on the gravitational background (2.7), except for the following three points. First, the coefficient of $\tilde{\mathcal{F}}_i^\alpha d_\alpha{}^\beta \tilde{\mathcal{F}}_\beta^i$ is $1 - W^{0\mu}W^0_\mu/(M^0)^2$ in refs. [13, 15]. However, this discrepancy is harmless because W^0_m is odd under the orbifold parity and vanishes on the boundary. Both actions lead to the same on-shell action.[23] Second, the coefficient of the Einstein-Hilbert term is different from that of ref. [13, 15], which is $e\mathcal{A}_i^\alpha d_\alpha{}^\beta \mathcal{A}_\beta^i/4$. This discrepancy stems from the fact that we have dropped the auxiliary field D that leads to the constraint (3.7). In fact, both coefficients will be the correct value $-eM_5^3/2$ after the gauge fixing. Third, according to refs. [13, 15], the coefficient functions of the kinetic term for the gauge fields and gauge scalars are $\mathcal{N}a_{IJ}$ defined in eq. (3.12) instead of $-\mathcal{N}_{IJ}/2$ in eq. (B.1). This stems from the fact that we have replaced the auxiliary field $v_{\underline{\mu\nu}}$ in the Weyl multiplet by its background value. However, this does not cause a serious problem if we do not discuss the graviphoton sector. In fact, after the gauge fixing, the difference from the correct one only affects in the quartic or higher order terms that are neglected here, except for the graviphoton sector. Therefore, eq. (2.16) effectively reproduces the invariant action in refs. [13, 15].

From eq. (B.1), we can obtain the on-shell expressions of $V_y^{(r)}$ and $Y^{I=0(r)}$ $(r = 1, 2, 3)$.

$$\begin{aligned}
V_y^{(3)} &= -\frac{1}{M_5^3}\mathrm{Im}\left(\bar{\mathcal{A}}_2^3\partial_y\mathcal{A}_2^3 + \bar{\mathcal{A}}_2^4\partial_y\mathcal{A}_2^4\right) + \cdots, \\
V_y^{(1)} + iV_y^{(2)} &= -\frac{i}{M_5^3}\left(\bar{\mathcal{A}}_2^3\partial_y\bar{\mathcal{A}}_2^4 - \bar{\mathcal{A}}_2^4\partial_y\bar{\mathcal{A}}_2^3\right) + \cdots, \\
Y^{0(3)} &= -\frac{M_5^2 g_{\mathrm{c}}^0}{3}\left(1 + \frac{\left|\mathcal{A}_2^3\right|^2 + \left|\mathcal{A}_2^4\right|^2}{M_5^3}\right) - \frac{g_{\mathrm{h}}^0}{3M_5}\left(\left|\mathcal{A}_2^3\right|^2 - \left|\mathcal{A}_2^4\right|^2\right) + \cdots, \\
Y^{0(1)} + iY^{0(2)} &= \frac{2g_{\mathrm{h}}^0}{3M_5}\bar{\mathcal{A}}_2^3\bar{\mathcal{A}}_2^4 + \cdots,
\end{aligned} \tag{B.3}$$

where the ellipses denote terms that have vanishing VEVs. We have used the gauge fixing conditions. $Y^{I\neq 0(r)}$ does not involve the stabilizer fields $\mathcal{A}_2^{\alpha=3,4}$, and have vanishing VEVs under the assumption of eq. (4.8).

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